

Exam. Code : 121203
Subject Code : 502192

B.A./B.Sc. 3rd Semester (Batch 2024-27/28) (CBGS)

MATHEMATICS
(Differential Equations)

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Solve $x^2y \, dx - (x^3 - y^3) \, dy = 0$. 10
(b) Solve : $\cos^2 x \, dy/dx + y = \tan x$. 10
2. Find the necessary and sufficient condition that the equation $Mdx + Ndy = 0$ (where M and N are functions of x and y with the condition that $\partial M/\partial y = \partial N/\partial x$ are continuous functions of x and y) may be exact. 20

SECTION—B

3. (a) Find the orthogonal trajectories of the family of conical circles $x^2 + y^2 + 2gx + c = 0$, g is parameter.

10

(b) If $d^4y/dx^4 - 4y = 0$, show that

$$y = C_1 \cos(ax) + C_2 \sin(ax) + C_3 \cosh(ax) + C_4 \sinh(ax).$$

10

4. (a) Solve :

$$d^2y/dx^2 - 4dy/dx + 4y = 8(x^2 + e^{2x}) + \sin 2x.$$

10

(b) Show that the system of confocal conics $(x^2)/(a^2 + \lambda) + (y^2)/(b^2 + \lambda) = 1$ is self-orthogonal.

10

SECTION—C

5. (a) Obtain the primitive and singular solution (if it exists) of the equation $xp^2 - 2yp + 4x = 0$.

10

(b) Solve and test for singular solution :

$$p^3 - 4xpy + 8y^2 = 0.$$

10

6. (a) Solve : $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = x + \sin x.$ 10

(b) Solve :

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos(\log(\sqrt{1+x})). \quad 10$$

SECTION—D

7. (a) Solve : $x^2 \frac{dy}{dx}^2 - 2xy \frac{dy}{dx} + 2y^2 - x^2 = 0.$ 10

(b) Solve: $4p^2x - (qx - k)^2 = 0;$ k is parameter. 10

8. (a) Solve :

$$\frac{dx}{[x(y^2 - z^2)]} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{[z(x^2 - y^2)]}. \quad 10$$

(b) Solve : $y'' + x^2y = 1 + x + x^2$ in powers of $x.$

10